present case, where u_t is the friction velocity, $C(n)$ is function of n only, its value having been tabulated by Schlichting[5] book, and ν is kinematic viscosity of the fluid. Employing the boundary condition at $y = \delta$, we can transform Eq. (5) into the form

$$C_f = 2C^{-2n/(n+1)}(\delta U/\nu)^{-2/(n+1)} \quad (6)$$

which gives a relation of C_f in terms of δ. After substituting Eq. (6) into Eq. (4) and integrating, we have

$$\left[\frac{n}{(n+2)(n+3)} + \frac{n}{2(2n+1)(n+2)}\frac{R_\delta}{R_a}\right] \times R_\delta^{(n+3)/(n+1)} = D(n) R_x \quad (7)$$

where

$$R_\delta = \frac{\delta U}{\nu}, \quad R_a = \frac{aU}{\nu}, \quad R_x = \frac{xU}{\nu}, \quad D(n) = C^{\frac{-2n}{n+1}}$$

For the case of flat plate $R_a = \infty$, Eq. (7) can be reduced to

$$R_{\delta_F}^{1.25} = 0.288 R_x, \quad (n = 7) \quad (8)$$

where the subscript, F, denotes the quantity for the case of flat plate. Dividing Eq. (7) by Eq. (8), we have

$$\left[\frac{n}{(n+2)(n+3)} + \frac{n}{2(2n+1)(n+2)}\frac{R_\delta}{R_a}\right] \times \frac{R_\delta^{(n+3)/(n+1)}}{R_{\delta_F}^{1.25}} = \frac{D(n)}{0.288} \quad (9)$$

Equation (9) represents a comparison between δ and δ_F at the same value of R_x. Assuming $n = 7$, we can reduce Eq. (9) into a simpler form

$$\delta/\delta_F = (1 + \delta/3a)^{-0.8} \quad (10)$$

which was obtained by Landweber.[6] It was shown by Joseph et al.[4] that n is no longer equal to 7 for cylinder radii $a < 0.75$ in. Equation (10) will not give accurate results for small cylinders. This part will be discussed later. For a given radius of a circular cylinder, n can be calculated by Eq. (3). Knowing the value of R_x, we can calculate $R_{\delta F}$ by Eq. (8), than the corresponding R_δ can be calculated by Eq. (9). Figure 1 is a comparison of the boundary-layer thickness calculated by the present method to the experimental data of Joseph et al.[4] Good results are obtained. Figure 2 shows the results for $a = 0.25$ in. Landweber's[6] and the present theoretical results are also shown. For this case $n = 10.1$. The present method shows better agreement with the experimental data than do Landweber's[6] predictions. Other boundary-layer parameters, such as displacement thickness and momentum thickness, can also be predicted by the power law relation.

Knowing the development of δ, one can easily calculate the local skin-friction coefficient by Eq. (6); results are shown in Fig. 3. For the case $a = 0.25$ in., $n = 7$, the calculated results are also shown. It is seen that there is about 20% difference between the two. Unfortunately, no experimental data were available to the authors, however, the tendency is seen to be reasonable.

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Asymptotic Suction Flow of Power-Law Fluids

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Introduction

SUCTION has been approved to be a powerful method for boundary-layer control for the purpose of increasing lift and reducing drag. Many theoretical as well as experimental studies have been reported. A surprisingly simple solution can be obtained when the velocity components are independent of the longitudinal coordinate. Among these solutions, Schlichting obtained a solution for the flow over a flat plate at zero incidence with uniform suction. Liu[2] obtained an unsteady asymptotic suction solution when the external flow is an exponential function of time. This Note presents a class of asymptotic suction solution for the flow of power-law fluids over a flat plate.

Basic Equations and Solutions

Under the assumptions of study and two-dimensional asymptotic suction flow, the momentum equation for the flow of power-law fluids over a flat plate can be expressed as

$$V_0 \frac{du}{dy} = \frac{d}{dy}\left(k\left|\frac{du}{dy}\right|^{N-1}\frac{du}{dy}\right) \quad (1)$$

with the boundary conditions

$$u = 0 \quad \text{at} \quad y = 0$$
$$u = U \quad \text{as} \quad y \to \infty$$

where y is the coordinate normal to the plate, V_0 and u are, respectively, the velocity components normal and along the plate. N and K are parameters related to power-law model.

Equation (1) can be integrated with respect to y, yielding

$$V_0(U - u) = -K\left|\frac{du}{dy}\right|^N \quad (2)$$

in which the constant of integration is determined by the condition that at $y \to \infty$, $u \to U$ and $du/dy = 0$. This can be integrated again by separation of variables, as evident

Received August 31, 1972; revision received February 7, 1973.

Index category: Boundary Layers and Convective Heat Transfer—Laminar.

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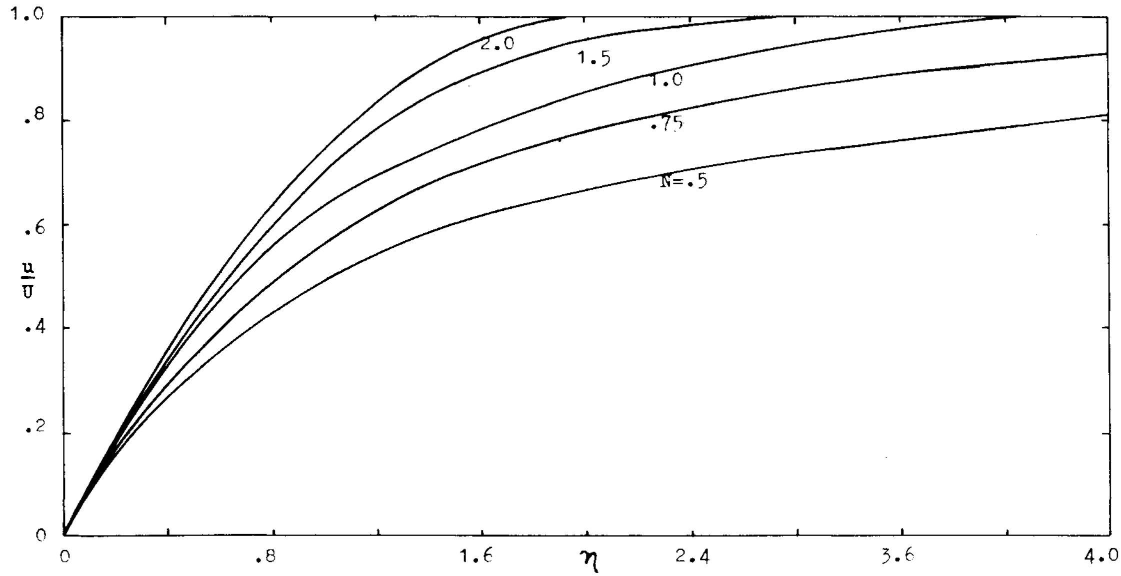

Fig. 1 Asymptotic suction velocity distribution.

if Eq. (2) is written as

$$\frac{du}{dy} = -[(\frac{-V_0}{K})(U-u)]^{\frac{1}{N}} \tag{3}$$

We obtain

$$y = -(\frac{-K}{V_0})^{\frac{1}{N}} \frac{N}{N-1}[(U-u)^{\frac{N-1}{N}} - U^{\frac{N-1}{N}}], N \neq 1 \tag{4}$$

in which the constant of integration is determined by the boundary condition $u = 0$ at $y = 0$. Solving this for u/U gives

$$\frac{u}{U} = 1 - (1-cy)^{\frac{N}{N-1}}, \quad N \neq 1 \tag{5}$$

where

$$c = (\frac{-V_0}{K})^{\frac{1}{N}} \frac{N-1}{Ny} U^{\frac{1-N}{N}} \tag{6}$$

Assuming

$$\eta = (\frac{-V_0}{K})^{\frac{1}{N}} U^{\frac{1-N}{N_y}}$$

Equation (5) can be written as

$$\frac{u}{U} = 1 - (1 - \frac{N-1}{N}\eta)^{\frac{N}{N-1}} \tag{7}$$

When $N = 1$, integration of Eq. (3) gives (putting $K = \nu$)

$$\frac{u}{U} = 1 - e^{\frac{-V_0 y}{\nu}} \tag{8}$$

and η becomes $-V_0 y/\nu$.

Figure 1 shows the velocity distribution for various value of N. The special case for $N = 1$ is also shown.

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